Felix Gelfenbein, Alla Shmukler

An extraordinary Math Lesson at the Dawn of the day

A mathematical fairy tale for children of senior preschool and junior school age.

Dedicated to my dear son and grandson Sasha in honor of his third birthday

Preface: the Place of Action

At the start of the 21st century, in the heart of the Arava desert in Israel, a source of underground water was discovered and named “Miracle Waters.” The land, or rather the sands, around this spring quickly gained significant value. The land belonged to the State and the State placed it at the disposal of the Ministry of Aliyah and Integration, aiming to establish there a settlement of a level and type that would attract wealthy repatriates from Western Europe and North America.

The goal was not to settle repatriates in the elite cities of the central part of the country, such as Tel Aviv, Caesarea, Herzliya, etc., but to direct them to the sparsely populated South of the country. The Ministry found construction firms and landscaping organizations which were entrusted with the task of creating an elite-type settlement around the source within a few years, including developed infrastructure, forest plantings and water facilities. The houses were to be built as two-story villas, separated from each other, using green areas as dividers. It wasn’t an easy task, and considerable resources were invested in its realization, but the project was completed on time and in full compliance with the settled goals.

By the end of the first decade of the 21st century, in the center of the former desert there was an oasis with a lot of green plantings, still young but rapidly growing.

Two-story mansions with flat roofs lined with solar boilers, television antennas and other means of modern communication were built in the midst of this magnificent greenery. The real wonder was the artificial lake in the center of the settlement, created using the natural

spring to which the settlement owed its appearance. The lake was filled with fish of different breeds, the purpose was not to breed them industrially, but to enjoy the pleasure of their contemplation. The name “Gan Eden” - “Garden of Eden” would have suited the settlement, but it was named after the underground spring, from which everything began: “Mey Nes” – "Waters of Wonder".

As is common, a massive advertising campaign of the new settlement was organized offering to buy houses of the project. In spite of the very high prices, there were many people who wanted to buy a house in the “Waters of Wonder” settlement. Priority was given to new repatriates, which received discounts while buying the houses in the settlement.

The Gross family was one of the last families to join the project. At the time, the family consisted of five members: Philip, or Phil Gross, 35, his wife Ann Gross, 30, their five-year-old daughter Alice, and two pets - a dog named Augusta and a cat named Thomas. The dog got its name due to its royal appearance – a soft and furry coat colored in yellow, noble features of a dog's face, elongated nose, wide-set gray eyes, thin lips hiding snow-white teeth.

Thomas the cat could be considered a black cat, if it were not for the white strip of fur on his neck, forming, so to speak, a white collar. In addition, there was a white spot on the forehead of Thomas.

Note that in contrast to the famous expression “live like a cat and a dog”, meaning a life full of quarrels and rejection of each other, Augusta and Thomas lived in complete friendship and harmony.

Augusta had a calm, balanced temperament, and even when Thomas jumped on her back, she did not throw him off, but calmly ran in circles with him several times waiting for him to come down.

The Gross couple were middle-income people, but Ann Gross had received a huge inheritance from her father, one of the first oil producers in North America. Therefore, she was able to make a worthy purchase in Israel and thus fulfill her long-held dream of moving from Canada, where they were born, raised a family and lived well, to live permanently in Israel.

The reader understands that in this case this decision was made for ideological, not material, reasons.

The Gross couple bought a home in Waters of Wonder and moved in, in the spring of 2008. At that time, their daughter Alice was five years old. Two years later, in the spring of 2010, Alice became a sister - a son was born to the Gross couple. The boy was named Moshe in honor of the great Jewish prophet. This name in English is pronounced Moses, in a diminutive form - Mosie. On the day when Moses turned 3 years old, an extraordinary event happened to him, which opened for him the door to a huge world previously unknown and invisible to him.

The creatures that inhabit this world are not human beings, but they are like them in many ways. They can perform various actions and enter various relations with each other.

They are diverse, differing in size and in character, but at the same time they all share one common property: they are meant to describe quantitative relationships in the human world.

These beings are called Numbers.

In our story we will open for you, our dear little readers and listeners, the door to this world - the world of numbers.

So, good luck on your journey!

1. Awakening

On one of the days in March, 2013, when the sun had just risen above the horizon and its first rays lightly touched the flat roofs of the houses in Waters of Wonder settlement, Mosie Gross woke up in his high-sided crib in his room on the second floor of the Gross family home. It wasn't a ray of sunshine that woke him. The dense blinds were down, and the gentle predawn ray of sunlight could not penetrate them.

Mosie woke up to a ray of light touching his eyes, coming from some source near his crib. Mosie woke up with a joyful feeling that had not left him since last night, even in his sleep. Last night his mother, putting him to bed and kissing him, had said that tomorrow, that is, today was Mosie's birthday, and on this occasion, there would be many guests, many congratulations and gifts. Perhaps, thought Mosie, there was some glowing present near the crib. He lifted himself up, stood in the crib, holding on to the wall, and looked into the place where the light was coming from. He saw a little girl who looked very much like his sister Alice, or Alisa, as Aunt Laura called her. This aunt lived in Germany and occasionally came to visit her brother Philip's family.

“Alice?” Mosie asked.

“No, I am not Alice,” said the girl, “I only tried to be like her so that you would not be frightened, when you'd see me."

“And who are you?”

“My name is Taura, or Tau for short.”

“Laura?” asked Mosie, remembering Aunt Laura.

"Not Laura, but Taura, or Tau. But you know what, it will be easier and more familiar for you to call me 'Fairy Taya'."

“A fairy?”.

"Yes, a fairy, because I am a fairy who can do all kinds of wonders. And my magic wand is this ray."

She raised her right hand and from under the fingernail of her index finger a ray popped out, the same ray that had woken up Mosie.

Next, Fairy Taya continued:

“To make you believe in my magic power, I will show you one of the miracles, not the biggest, a small miracle.” Just then a ray of light touched Mosie's hair. Suddenly he felt himself growing uncontrollably. Earlier he had been looking at the girl through the bars in the headboard, but now he was already towering over the headboard.

His feet remained in the crib, while his head rose higher and higher until it touched the ceiling. Then the magical ray touched his hair again, and he gently descended back into the crib, returning to the former little Mosie. Everything happened so quickly that Mosie only had time to say, "Oh!"

Then the back of the crib lowered by itself, and Mosie found himself sitting on the edge of the crib. His feet touched the floor, and his eyes looked directly at the girl.

"Well, now you don't doubt that I am a fairy, do you? So just call me 'Fairy Taya.'"

"Fairy Taya, how did you find yourself in my room?" Mosie asked.

"I have been in it continuously for three years since the day you were born, or more precisely, since the day of your 'Brit Milah.' * I was brought here by your sandak **, Moshe Klein, who instructed me to stay with you and guard you for three years, until your third birthday. So today, I am leaving, and that's why I decided to reveal myself to you."

"But why have I never seen you before?"

"You couldn't see me because I was in my original, so to speak, natural size. And that size is such that no human eye, even with the strongest microscope, can see me. The thing is, my dear Mosie, that I am a number, an extraordinary one, smaller than any number visible to the human eye. Mathematician Moshe Klein discovered me theoretically in his research and decided to give me to you temporarily on the day of your 'Brit Milah.' Now that time has ended, and I must leave you."

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*Brit Milah (circumcision) – a ritual in Judaism performed on a boy on the eighth day after his birth.

**Sandak – the man who holds the child during the performance of the 'Brit Milah' ritual.

2. Where are you from, Fairy Taya?

We present here the story of Fairy Taya about her homeland. It should be noted that all conversations between Fairy Taya and Mosie are conducted in English. While secretly staying in the Gross family, Fairy Taya learned that the native language of this family is English. Although the adults and even Alice had a good command of Hebrew, they preferred to speak English. For Mosie, it is currently the only language of communication. Only now, at the age of three, will he attend kindergarten in the settlement, become acquainted with Hebrew, and start speaking it.

And so, Mosie is sitting on the edge of his bed with the front side lowered, listening to Fairy Taya's story. "Perhaps you do not yet know, my dear Mosie, that the entire space containing all that exists, the so-called Universe, is divided into separate enormous sections, much like a house is divided into apartments and apartments into rooms. These unimaginably vast sections of the Universe are called galaxies. Your planet Earth is situated in one of them. I, however, was born in another galaxy, very far from yours, but the closest to the One Who Created All that Exists. My galaxy is inhabited not by beings similar to humans, but by numbers, of all sorts, big and small, numbers of all types found in the Universe. The name of this galaxy is 'Numeratica', and it is constantly filled with new types of numbers, which then spread throughout the Universe. Numbers are born in the boundless mind of a being named Galapher. He received his powers from the One Who Created and Controls everything. The life of numbers is governed by the great science named Mathematics, and the laws that rule their lives and influence all existence form a profound teaching named Numerology. The combination of the two names 'Numerology' and 'Mathematics' gave name of my galaxy – 'Numeratica'. I was born in Numeratica relatively recently, when Galapher decided to create a new type of numbers, so small that no creature in the Universe can see them with their eyes. They can only be 'conceived,' that is, understood with the mind. Despite this, I am more than that number which everyone knows and depicts as a circle with emptiness inside. The meaning of this number is absolute zero, complete nothingness. The insignificance of zero, its weakness, is evident when it tries to unite with any other number that is not zero. Zero clings to the non-zero number, embraces it with all its might… and can do nothing to it; the number does not change at all from its embrace.

Against this background, the great power of zero seems even stranger when it wants to overpower a second number by multiplying it by itself. Then it simply swallows this number, turning it into absolute zero, into nothingness. When Galapher created me, he decided to marry me off to Absolute Zero, strictly forbidding him from fighting with me or multiplying me by himself. He was only allowed to embrace me, trying to merge with me. In this merging, we became a single number, which was me.

As a result of this union, we had countless sons, resembling their father in some ways and me in others. We named them 'Soft Zeros.' From me they inherited a size invisible to the ordinary eye.

When fighting with a regular number, a soft zero, unlike its father, does not completely swallow it, that is, does not turn it into itself, but only increases its size at the expense of the other number. But if two soft zeros start fighting each other, that is, multiplying by each other, they both disappear, returning to the bosom of their father, so that in place of two soft zeros, there appears one absolute zero. Neither I nor my husband – Absolute Zero, wanted our children to disappear, so we built a separate house for each of them, and these houses filled an infinitely long street, at the center of which stands a large house where we live – my husband Absolute Zero and I, the extraordinary number Tau. I am larger than zero, which somewhat offends my husband, but like him, I am smaller than any positive number, which somewhat consoles him. He surpasses me in one aspect: being essentially nothing, he is nevertheless perceived visually.

The street in the galaxy Numeratica where our house is located, with the houses of our children on either side, is called 'Zero Avenue.'

From time to time, we leave our house to interact with other numbers in the surrounding space. My husband is avoided because others fear being multiplied by him, while I and our children, the soft zeros, remain unseen. I can see them well, and it is my duty to prevent them from colliding with each other, as such collisions could lead to their disappearance, as I mentioned earlier.

This was our life for quite a long time after our union and the birth of our children. Then something happened. When my children grew up and started venturing out of their houses, they met other numbers there, those that had already spread throughout the Universe and are called real numbers.

As I mentioned before, soft zeros were forbidden from fighting each other. I strictly ensured this rule was followed. But they could freely interact with other numbers – unite and even fight. From these interactions, a new type of number emerged, combining real numbers and soft zeros. These numbers were called 'Soft Numbers', and they quickly filled the entire galaxy of Numeratica, which Galapher then renamed to 'Ephraxia.'

I should tell you that the language in our galaxy is neither English, nor Russian, nor Spanish, nor any other language common in your world. Our language is Hebrew because it is the language of Creation. The words of God that created the Universe were spoken in Hebrew. Regarding numbers, some of them, the so-called natural numbers, originated during the creation of the Universe. They were used to denote the days of Creation. The word 'Ephraxia' is derived from two Hebrew words: 'Efes' – zero and 'Rach' – soft.

Since the soft zeros produced the new numbers called soft numbers, our galaxy, filled with these numbers, was named 'The Galaxy of Soft Zeros,' or in Hebrew – Ephraxia.

When Ephraxia was finally formed, Galapher decided to introduce the inhabitants of other galaxies to the soft numbers. He sent representatives of this category of numbers throughout the Universe to inform the intelligent beings of other galaxies about these new numbers.

I was sent by Galapher to the intelligent inhabitants of your galaxy, living on the planet Earth. Here, I stayed for some time in my usual invisible-to-the-human-eye state. However, after a while, one of the Earth mathematicians named Moshe Klein managed to perceive me intellectually and make contact with me. I told him about the soft zeros and soft numbers, and he then presented this knowledge in his doctoral dissertation and in scientific journals. This way, the people of Earth were able to learn about the inhabitants of Ephraxia. Then my mission was complete, and I could return home. I wanted this very much, as I missed my husband and children. But Moshe pleaded with me to stay for another three years in the Gross family's house to look after you and then give you your first math lesson. I was grateful to Moshe for dedicating his life to spreading knowledge about me, my children, and all the soft numbers since meeting me. Therefore, I agreed to stay for three years with the Gross family. None of the family members could see me because I was in my usual size, undetectable to the human eye. But I was always near you and saw everything. When you were sitting on the lawn near the house and a wasp was about to sting you, I drove it away with my magic beam. There was a much more serious case, when a venomous snake slithered from the desert sands to your house and coiled around your leg. I had to kill it with the same beam. There were

many other instances when your life was in danger, and I prevented any possible harm. But today, on the day you turn three, I must part with you. However, before I go, I have to fulfill another promise I made to Moshe – to give you your first math lesson. For this, we will need a table and colorful pebbles". Having finished her speech, Fairy Taya raised the index finger of her right hand, and a beam slipped out from under her fingernail, drawing a circle in the air. Inside this circle, a horizontal table appeared, as if suspended in the air, with a pile of yellow and blue pebbles in one of its corners. These colors were well known to Mosie: blue was the color of the sky over the settlement Waters of Wonder, and yellow was the color of the sands surrounding the settlement. "Well, now everything is ready, and we can start the lesson."

3. The Lesson

The lesson taught by the Fairy Taya to Mosie took place in the form of a dialogue: the Fairy Taya explained and asked something, and Mosie answered and carried out her tasks, sometimes also asking questions. We reproduce this dialogue as it occurred, prefacing the words of Fairy Taya with the underlined heading 'Fairy Taya', and the words of Mosie with the heading 'Mosie'. This dialogue is occasionally interrupted by headings with discussed topics, introduced by us for the convenience of the readers. The heading below marks the first discussed topic.

What is a natural number

Fairy Taya - Now I will introduce you to the world of numbers. Numbers are found in every corner of the Universe. Sometimes they are invisible, and sometimes they are hard to detect. There are many kinds of numbers, but the most important number, from which all other numbers originated, is the number "one". It is depicted differently in various places and at different times, but its meaning is always the same. We say "one" when we see an object appearing alone, by itself, without being accompanied by other similar objects. Look, observe, and tell me what you see.

Mosie

You waved your hand, and a small cube appeared before me, it is just hanging in the air, without any support. Now another cube joins it.

Fairy Taya

Now there are two cubes. This is the number "two". Here, I am adding another cube to them, now the total number of cubes is "three". Now I am adding another cube, and before us are "four" cubes, I add another one, and now the total number of cubes is "five". Tell me, what do you see?

Mosie

I see five identical cubes hanging in the air, without any support.

Fairy Taya

In this way, we can continue further and obtain more and more numbers. We see that all these numbers are obtained by adding the number "one" to the previous number. That is why I said that the number "one" is the most important, fundamental number. Moreover, the number "one" existed even before the beginning of the creation of the world, because then there was nothing and no one except the Creator. He was One, and during Creation, all other numbers arose, which are obtained using the number "one", as I have shown you. There are infinitely many such numbers, and they are called natural, meaning counting numbers. With the help of these numbers, man, the crown of Creation, discovered many other numbers. We will not talk about them today. We will only get acquainted with some natural numbers. Essentially, you already know the first five natural numbers. Some of them live on your body. I see the surprise on your face. Do not be surprised, now you will see for yourself that it is so.

Numbers on the human body

Let's start with the most important number – "one". You have one head, my little beam touches it and ruffles your hair. The head is the main organ of your body. Thanks to it, you think and understand other people. You have one forehead, one nose, one mouth, and you have one organ, which you currently use for only one thing, and which

your mother calls "pee-pee". But in the future, it will ensure the continuation of your family line.

Let's turn to the number "two". It also lives on your body. You have two eyes, two ears, two eyebrows, two nostrils and two lips. And this is all on your face.

Now let's move lower. You have two hands, here they rise up, and also two legs. Here they spread slightly apart.

An important number living on your body is the number "five". You have five fingers on each hand and five toes on each foot.

Now, raise your right hand. We will count your fingers. Just keep your hand raised, look at your fingers, and concentrate on them: after I name a finger, it bends.

One – thumb, two – index finger, three – middle finger, four – ring finger, and five – little finger. And now all your fingers have formed one fist. Open it and place the palm of your left hand against the palm of your right hand, finger to finger: little finger to little finger, ring finger to ring finger, middle finger to middle finger, index finger to index finger, thumb to thumb. Raise the joined palms up. Now it is clear that on the left hand there are as many fingers as on the right, exactly five. And five and five together make the number ten. A person has exactly ten fingers on both hands.

The number "ten" is the second most important number after the number "one" among the numbers living on the human body. Since this number lives on the hands, and the hands are the most active organ of our body, people in ancient times used the number "ten" to determine the values of other numbers. In other words, they counted by tens. This counting system, of course in its improved form, is used up to our time.

One number for different groups of objects.

In general, Mosie, every natural number except for "one" is associated with a group of objects. Moreover, groups of completely different objects can correspond to the same number. Look here, Mosie, in front of you is a group of cubes that you have seen before, below it is a group of balls, and below the balls there is a group of flowers... Do you see them, Mosie?

Mosie

Yes, I see them, they are hanging in the air, one below the other**.**

Fairy Taya

There are three groups of different objects, because a cube is not a ball, and a ball is not a flower. But these three different groups share one common property – the number of objects in each of them is described by the same number – 'four'. This number is represented by each of these groups. We will use groups of pebbles of different colors to represent numbers. Now look and tell me what do you see?

Mosie

In front of me suddenly appeared and hung in the air a board resembling a chessboard, but without squares. On the board was a pile of pebbles of two colors – blue and yellow.

Fairy Taya

This group of pebbles corresponds to a certain number. We won't determine it now, but we will break it down into two another numbers. Just watch and tell me what you see.

Mosie

Your beam slides over the pile of pebbles, causing the blue pebbles to line up in a row and the yellow pebbles to form another line below the blue ones.

Fairy Taya

The group of yellow pebbles corresponds to one number, and the group of blue pebbles to another. Together, these numbers add up to the number that corresponded to the group of pebbles of both colors. We have just broken down one number into two numbers. With the help of these pebbles, we will learn to perform some other operations on numbers as well. But first, let's return to the numbers that live on your body. Let's look at the number that lives on your head. Here, my beam is ruffling your hair, and it stands on end. But now it smooths your hair down, and it lies flat again, as if you have just combed it.

About large numbers and the largest number - does it exist?

The number of your hairs is enormous, it can be counted, and this number has a name. We won't do that now, but it's a very large number. Know, my dear Mosi, that numbers have no end. There is no such thing as the largest number in the world. No matter how large a number you name, there is always a number even larger than that.

Now let's talk about the numbers that surround us. First, we will learn to determine quantities of relatively small numbers, using counting by tens.

How to count by tens

Here is a group of pebbles arranged in a single line in front of you. Set aside five pebbles from the beginning of the line. Then set aside another five pebbles, making a total of ten pebbles. After this set of ten, leave a small gap, and in the remaining group of pebbles, set aside another ten, and so on. Follow this rule and tell me what you are doing.

Mosie

I do as you said: I count out five pebbles, then another five, making a total of ten. There are still many pebbles left. From these, I count out five and another five. Again, that's ten. From the remaining ones, I count out five and another five. And now there are only two pebbles left...

Fairy Taya

Good job, Mosie, you did everything correctly. Now we know the number of pebbles: three tens and two more pebbles. You know what, open your mouth. I'll take a look inside. You haven't grown all your teeth yet. But when you have all your teeth, there will be exactly as many as there are pebbles here: three tens and two, or simply "thirty-two." So, the number "thirty-two" will be present on your body.

What can be done with numbers without knowing their values

Now we will talk about sufficiently large numbers that we may or may not want to count. What can we do with them without knowing their values and without counting them by tens, but simply knowing that they are some numbers?

Look, here in front of you on the table there is a new pile of pebbles. Now they are not blue and yellow as before, but purple. A pile of purple pebbles. I don't know how many there are. But we will now learn a lot about their number without knowing its exact value or name. For this, we need to arrange all the pebbles in a line. I will do this with the help of my little beam. Here it touches the purple pebbles, and they immediately line up one after another.

Now, Mosie, you see a line of pebbles representing a certain number. We will study the properties of this number without knowing its value or name. The properties of a number are its internal qualities, not related to its name. By the way, the names of the same number, my dear Mosie, are different in different languages, but the essence of the number is the same and does not depend on the language or the place where the number is found. To find out the properties of the number represented by the group of purple pebbles, let's try to divide this group into equal parts. First, into parts with one stone each. Make sure there is a space between each stone.

Did you understand everything?

Mosie

Yes, I understand. And I'm already doing it very quickly, as always when I understand what is expected of me.

Fairy Taya

Yes, Mosie, you've done it quickly and correctly. Now the entire group of pebbles has been divided into equal parts, each with one stone. This means that the total number of pebbles is divisible by "one".

And now, Mosie, look: the pebbles have miraculously lined up in a single line again without any gaps. Let's try to divide the number they represent by "two". Take two pebbles, leave a

space after them, then take another two pebbles and leave another space, and so on. This way, we will divide the number of pebbles by "two". Let's see what happens...

Well done, Mosi. I see that you're doing everything just as I said, and very quickly. You're a clever boy. Now, tell me what you did and what you found out.

Mosie

First, I set aside two pebbles, made a gap, then another two, another gap, and then another two, and repeated this many times until there was only one stone left without a pair. I ended up with many pairs, and then one stone left without a pair.

Fairy Taya

You see, Mosie, we ended up with one stone left over. In this case, the number we divided is called odd. Now we know that the number corresponding to our group of pebbles is an odd number.

Even and odd numbers

Mosie

And if there were no remainder, and all the pebbles were paired, would the number be divisible by "two"?

Fairy Taya

Well done, Mosie, asking the right question is very important! I'll answer it. A number that can be divided by "two" without a remainder is called even. Every number representing the quantity of objects in a group is either even or odd. The number of purple pebbles turned out to be odd. In other cases, there could be an even number of pebbles. What do you think, dear Mosie, if I have one pebble in my hand? Look, I have one pebble in my palm. But it's not purple - it's orange. If I have the number "one" in my hand, do you think it's even or odd?

Mosie

I don't know ... Well, here you can't make a pair...

Fairy Taya

Yes, exactly, so there are no pairs at all here, only the remainder right away. Therefore, the number "one" is an odd number.

What do you think, what kind of number is "three"?

Mosie

Three pebbles... Well, you can only make one pair out of them, and then one pebble will remain. You can't make a pair out of it; it's a remainder. So the number "three" is odd.

Fairy Taya

Good job, Mosie! You've grasped this concept. So, you remember that the number of items in a group is either an even or an odd number. Now, let's talk about some properties of numbers that can be determined without knowing their values.

Prime and Composite Numbers

Let’s return to our pebbles. We know that their number is odd. So, it is not divisible by “two.” Now, let’s try to divide this number by “three.” Make groups of three pebbles. Leave a gap after each group.

Mosie

I understand what to do: I count three pebbles, then a gap, then three more pebbles, a gap, and so on… There are many groups of three pebbles. And finally, there is a remainder — one pebble.

So, is this number divisible by “three”? No, it isn’t. It isn’t divisible because there is a remainder. When I divided it into groups of three pebbles, there was a remainder, so it isn’t divisible.

Fairy Taya

See, it’s not divisible! Now, let’s try to divide this number by “five,” that is, let’s try to divide the entire group of pebbles into parts of five pebbles each.

Mosie

Now I count five pebbles, then a gap, then five more pebbles, and another gap, and I continue until it is possible… And here’s the end—only three pebbles are left.

Fairy Taya

See, Mosie, it’s not divisible by “five” either. Let’s try to divide it by seven. Maybe you don’t know what seven is? I’ll tell you. Five and one is six, and five and two is seven. Make groups of seven pebbles with a gap between them and keep going till the end of pebbles… I see you’ve started doing it, continue till the end of pebbles and tell me what will you get at the end.

Mosie

I finished and got the remainder of one pebble.

Fairy Taya

So, it’s not divisible by seven either. Now, dear Mosi, let’s try to divide it by nine. What is the number “nine”? If we count from seven: seven and one is eight, and eight and one is nine. Look, I’m counting nine pebbles and making a gap after them, and you will continue.

Mosie

I count nine pebbles—done, a gap, now nine more pebbles, a gap, nine more and a gap, but I can’t go further, I’m missing two pebbles.

Fairy Taya

See, it’s not divisible by nine either. How many pebbles are left? Count them.

Mosie

One, two, three, four, five, six, seven pebbles—seven pebbles are left.

Fairy Taya

Even if we continue this work and try to divide the entire group of pebbles into equal parts containing more and more pebbles, but fewer than the total number of pebbles, you'll see that our number isn't divisible into equal parts, except into parts containing one pebble each. Know, Mosie, if a group of objects cannot be divided without a remainder into any equal parts except parts of one object, the number representing this group is called a "prime number." So, the number of pebbles in our group is a prime number. Prime numbers were discovered by ancient people. Imagine a primitive man collecting shells and pebbles on the seashore and playing with them. Especially if this primitive person was still young—a boy or a young man. How did he play with the pebbles? He tried to divide them into equal parts. Sometimes he succeeded, and sometimes he didn't. Thus, prime numbers were discovered in ancient times. Prime numbers are very important numbers. Why? Because, they are like bricks that are used for building a house, we can build all natural numbers from prime numbers. How? You'll find out in the future. But now you know what a prime number is. All numbers representing the quantity of objects in a group are divided into prime and non-prime numbers. Non-prime numbers are called composite. Now, Mosie, tell me, is an even number prime or composite?

Mosie

Even… Oh, yes, it's when we make a group of pairs. So, it can be divided by "two." So, an even number isn't prime, it's composite.

Fairy Taya

But what about the number "two"? A group of two pebbles—into what equal parts containing less than two pebbles can it be divided?

Mosi

Well, that's so simple: into two parts, each containing one pebble.

Fairy Taya

So, the only number less than "two" that "two" can be divided by without a remainder is "one." So, is the number "two" prime or composite?

Mosi

…It seems to be prime.

Fairy Taya

Excellent, Mosi, you're right. The number "two" is a prime number. It's an even and at the same time a prime number. All other even numbers are composite. The even number "two" is a prime number. And what do you think, Mosi, are there any other even prime numbers besides "two"?

Mosi

It seems to me that all other prime numbers are odd.

Fairy Taya

Yes, a prime number, except for "two," is necessarily odd. But there are also odd numbers that are not prime. We'll see this now… Well, Mosi, now something new has appeared before you. Tell me, what exactly and how did it appear?

Mosie

I saw a magical beam appears, glides across the board, and a line of red pebbles emerged on it.

Fairy Taya

Let's try to count them in tens. You've done this before. Do the same and tell me everything you're doing.

Mosie

I count five and another five, it makes ten. A gap. Five and another five makes ten again. A gap. Five and another five makes ten again. A gap, and that's all. Because there are three pebbles left. I can't make a group of ten pebbles from them. So, there are three tens and three pebbles in total.

Fairy Taya

What do you think, Mosie, is this number even or odd?

Mosie

To find out, I need to try to split all the pebbles into pairs. Here's the first ten, I split it into pairs, and I did it without a remainder. Now the same with the second ten… and with the third… I made many pairs and three pebbles are left. I can make one pair from them, but one pebble is left, which cannot make a pair.

Fairy Taya

So, is the number of all pebbles even or odd?

Mosie

This number is odd.

Fairy Taya

But what do you think, Mosi, is it a prime number?

Mosie

It's not divisible by two, of course, the entire group of pebbles can be divided into parts of one pebble each, but can it be divided into any other equal parts? ...Oh, if I take one pebble from each ten, I get one group of three, and the remaining nines of pebbles can be divided into threes. So, the total number of pebbles is divisible by three!

Fairy Taya

Well done, Mosie, you'll be a mathematician! Or someone who loves math and uses it in his work. So, this number can be divided by three. And that means it's not prime. Because, besides being divisible by "one," it can be divided without a remainder by "three." This doesn't mean it can't be divided by other numbers. But if it can be divided by at least one number that is less than it and greater than "one," then it's not a prime number. It's a composite number. So, the number of red pebbles is a composite number. Although you don't know its name yet, you already know it's not prime but composite. And now, Mosie, what do you think, is the number one a prime number or a composite number?

Mosie

Well, it’s very simple: if I have only one pebble, it can’t be divided into equal parts containing fewer than one pebble. So, the number “one” is a prime number.

Fairy Taya

Mosie, your answer is based on the rule for determining a prime number. But there are exceptions to every rule. Know, Mosie, that the number “one” is not considered prime or composite by general agreement. It’s a special number, the beginning and basis of all numbers, and therefore it is an exception to the general rule.

The connection between even numbers and prime numbers is a question over 280 years old.

As we've already figured out, Mosie, any prime number except for 'two' is necessarily an odd number. No other even number can be prime, but there is a relationship between even numbers and prime numbers. Specifically, it is conjectured that any even natural number greater than 'two' is the sum of two prime numbers. For example, the even number 'eight' is the sum of two primes, 'three' and 'five'. The even number 'ten' is the sum of 'three' and 'seven'. And the even number 'six' is the sum of two identical primes, 'three' and 'three'. Similarly, 'six' is also the sum of two different numbers, 'one' and 'five', but as we've mentioned, 'one' is not considered a prime number. Therefore, the unique prime factorization of 'six' into two primes is as previously mentioned, 'three' and 'three'.

The conjecture that every even number starting from 'four' can be represented as the sum of two primes has been verified for many even numbers but remains unproven for all. Although it was proposed long ago, nearly three centuries ago. If you ever find such a proof, you will be renowned worldwide.

And now, dear Mosie, we are moving on to another topic of our lesson. This topic is a comparison between two numbers.

How can you compare two numbers without knowing their magnitudes?

In this part of the lesson, you will learn to compare two numbers without knowing their magnitudes.

The relationship between two numbers can be similar to the relationship between two boys in terms of height. Do you have many friends, Mosie? Tell me, who among your friends is taller than you? Do you know anyone like that?

Mosie

Of course, I know: Daniel is taller than me. He's a whole head taller, even though he's not much older. He's only three years and a few months old.

Fairy Taya

Do you know anyone among your friends who is shorter than you?

Mosie

Well, there are little ones, really tiny. Our neighbors have two-year-old Ethan. When he stands next to me, though he can barely stand, I'm definitely much taller than him.

Fairy Taya

Now, Mosie, here's the toughest question: do you have any friends who are as tall as you? When you stand next to them, it's like they say, 'of the same height'.

Mosie

I can't seem to recall anyone with whom I would truly be equal in height... Maybe except for Baruch. He and his parents moved here recently, and this boy used to be called Berthold, but now they call him Baruch. When we stand next to each other, one of us may be just slightly taller. I think, though, that we are about the same height.

Fairy Taya

Well, with numbers, there's no concept of 'just a bit'. Between numbers, there are only three types of relationships: one number is either greater than the other, less than the other, or they are equal. One number can't be slightly greater than another number; if it's greater, it's simply greater. If it's slightly less, then it's less. Or they are exactly equal. And now we'll learn how to determine this.

Look, in front of you will be two numbers. This means two piles of pebbles of different colors. Here, I'll send out my beam, and it will create something new on the board. Tell me what you see.

Mosie

I see pebbles of blue color appearing on the board, blue like the sky. Beautiful, beautiful blue pebbles. They are arranged in a single line, and below them, a line of yellow pebbles has appeared. Two different lines of pebbles - yellow pebbles beneath the blue ones, like sands under a blue sky.

Fairy Taya

Mosie, can you determine if there are more blue pebbles or fewer yellow pebbles, or perhaps if the number of blue pebbles is equal to the number of yellow pebbles?

Mosie

I can't say for certain right away. It seems there are more yellow pebbles than blue ones, or maybe there are more blue pebbles?

Fairy Taya

Now we will apply the appropriate method and accurately determine which color of pebbles there are more of. We will align the blue pebbles with the yellow ones so that under the first blue pebble there will be the first yellow pebble, under the second blue pebble there will be the second yellow pebble, and so on.

So, what do you see now?

Mosie

I see two lines of pebbles arranged as you described. Under the last blue pebble, I see a yellow pebble, but it's not the last among the yellows. There are two more yellow pebbles after it.

Fairy Taya

Now tell me, Mosie, which color of pebbles do you think there are more of?

Mosie

Of course, there are more yellow pebbles. The yellow pebbles under the blue ones are the same number as the blue ones. But there are two more yellow pebbles that do not have blue ones above them. Therefore, the number of yellow pebbles is greater than the number of blue pebbles.

Fairy Taya

If you remove these last two yellow pebbles, the ratio between the number of yellow pebbles and blue pebbles will change. Now, the number of yellow pebbles will be exactly equal to the number of blue pebbles, because each blue pebble has a corresponding yellow pebble underneath it.

Mosie

These numbers are equal. The number of yellow pebbles is now equal to the number of blue pebbles.

Fairy Taya

Okay, Mosie, what should I do if I want the number of blue ones to be less than the number of yellow ones again?

Mosie

Well, for example, you'd need to remove three blue ones.

Fairy Taya

So be it. Here, my beam removes the last three blue pebbles. Now the line of yellow pebbles continues under the blue ones exactly three pebbles further. Now there are more yellow pebbles than blue ones, and exactly by three.

And now, my dear Mosie, we have come to the final topic of our lesson. You already know that among natural numbers there are different types: even, odd, prime, composite. And now I will introduce you to one more type of natural numbers.

Perfect numbers

Now there's only one group of pebbles in front of you, and they are all blue. Come on, count how many there are.

Mosie

One, two, three, four, five, six. There are six pebbles in total.

Fairy Taya

And now, Mosie, we will divide this group of pebbles into equal parts, each containing fewer pebbles than the entire group. For each division, I will create a group of six pebbles of a specific color.

Here in front of you there is a group of six red pebbles. Divide this group into equal parts with one pebble in each part, with a gap between them.

Mosie

I'm doing this very quickly. Already done.

Fairy Taya

Now here is a group of six orange pebbles. Divide this group into parts of two pebbles each. Mosi, how many parts are there?

Mosie

There are exactly three parts.

Fairy Taya

Now this is a group of six yellow pebbles, divide it into equal parts of three pebbles each. How many parts are there?

Mosie

There are exactly two parts.

Fairy Taya

Notice, Mosie, that in all these divisions there was no remainder. Now, what happens if you try to divide a new group of six green pebbles into equal parts of four pebbles each?

Mosie

There is one group of four and two green pebbles left over.

Fairy Taya

And what if you try to divide this group of six blue pebbles into equal parts, with five pebbles in each?

Mosie

There is one group of five and one pebble left over.

Fairy Taya

So, Mosi, we divided the number six by all numbers less than itself. Now, Mosie, let's focus only on those divisions where there was no remainder. Here, I remove the groups of green and blue pebbles. Take the first part of the red pebbles obtained in their division, do the same with the orange and yellow pebbles. Then combine all the selected pebbles into one group. When you're done, I'll tell you what to do next.

Mosie

I've finished, done everything as you said: I took one red pebble from the line of red pebbles, two pebbles from the line of orange ones, and three from the line of yellow ones.

Fairy Taya

Excellent, you did everything correctly. Now I leave on the board only the original group of blue pebbles and the group of multicolored pebbles that you just built. I also arrange them in a single line and paint them in one color - violet. Now, what do you see on the board?

Mosie

I see two lines of pebbles: one line of blue pebbles and the another line of violet pebbles.

Fairy Taya

And now, Mosie, tell me how many violet pebbles are in the second line, not counting them?

Mosie

There are exactly six violet pebbles.

Fairy Taya

Why did you decide that?

Mosie

Because under each blue pebble in the first line, there is a violet pebble in the second line, and above each violet pebble in the second line, there is a blue pebble in the first line. I know that in the first line, the line of blue pebbles, there are six pebbles, so in the line of violet pebbles there are also six.

Fairy Taya

Excellent reasoning, Mosie, well done! You have learned the rule of comparing two numbers represented by groups of objects.

Now remember what you did. First, you divided the group of six pebbles into equal parts, each smaller than the entire group. Then from those divisions where there was no remainder, you took one part and combined these parts into one group. It turned out that this group of pebbles of three different colors, which turned into a group of violet pebbles, represents the same number as the original group of blue pebbles. And now listen to me carefully, Mosie:

If a natural number is divided without remainder by some other natural numbers smaller than itself, and the sum of all such smaller numbers equals the original number, then this number is called a perfect number. The number six is a perfect number.

So, we have established that the number six is a perfect number. Among natural numbers, there are many, even infinitely many, perfect numbers, but in the first ten, there is only one perfect number - "six". In the second ten, there are no perfect numbers, and in the third ten,

there is one. I suggest you to discover this number by yourself, Mosie. The actions needed for this discovery include dividing the number by a smaller number and comparing two numbers. You have learned to do this with the help of groups of objects. In our case, these were pebbles, but instead of them, there could be other objects, such as balls or sticks. I know that in your kindergartens and schools, sticks called 'bdidim' are used as objects for building groups representing numbers.

So, Mosie, I leave you with homework - to discover the second perfect number, located in the third ten. I'll give you a hint: this number is even. When you find it, present it as the sum of two prime numbers. I'm sure you can handle my task. And when it happens, we'll meet again, so I don't say 'goodbye', but 'until we meet again, my dear ward!' In the meantime, make friends with your 'sandak' and learn many interesting things about numbers. Now I will remove my guardianship from you.

4. A New Day - A New World

At that moment, Mosie looked around. What fairy? What board?

He's lying in his crib, and its side is raised. The room became quite bright, he can see all the objects.

But what's this?! He lifted his head, looked, and saw - a chandelier is hanging. It's one, it has no pair. Under it is a table. It's one, it has no pair. But around the table he noticed four chairs. Oh, two pairs of chairs!

Now all this rushed through his mind, although before he hadn't thought to notice any numbers.

He thought: "Where is fairy Taya?" He remembered everything that had happened, but thought that he was dreaming. Maybe nothing like that had happened, and it was just a dream.

And then the door opened, and mom came in. She was dressed more festively than usual, and she had such a solemn look. She came up to Mosie, hugged him and said: "Happy birthday, dear son! You didn't forget that today is your birthday, did you?"

Mosi immediately remembered about his birthday. He had remembered it yesterday. But after his conversation with fairy Taya, he had somehow forgotten about it. And now he remembered again.

"It's time to get dressed, come downstairs! Everyone is waiting for you there! And gifts have been prepared for you. You can't even imagine what they are! All of us, all of us have prepared gifts for you."

Mom went to the window and raised the blinds. The room was flooded with sunlight. Morning had fully arrived.

Mom said: "Let's go downstairs quickly. They're waiting for you there to congratulate you and present the gifts, you can't imagine what a wonderful gift Augusta and Thomas have prepared for you. Yesterday they spent almost the whole day in the sands, outside our settlement. There, Augusta dug a deep hole in the sand with her paws for a long time, Thomas jumped into the deepening pit and helped her from there. This continued until they found a shell glowing with an extraordinary light in the darkness, one of those that remained when the sea that once was here retreated from these places, and sands came to replace it."

Mom was saying all this while Mosi was already sitting on the edge of the crib, and mom was taking off his night clothes to replace them with festive ones hanging on a chair by the table.

And Mosie at this time was again remembering everything that had happened to him at dawn and thought that it had all been a dream.

But then mom approached the table to take Mosie's festive clothes off the chair, and suddenly exclaimed: "My God, has Aunt Linda already been here and left her expensive gift?!" On the table there were six small perfectly cut diamonds in the form of pebbles - three blue and three yellow ones.

"We'll need to call her as soon as possible and ask how she delivered her gift."

Aunt Linda, a wealthy relative of the Gross family, owned shares in the diamond exchange in Ramat Gan and owned a jewelry store in the center of Tel Aviv.

Mosie didn't respond to mom's exclamation, but he understood that everything that had happened to him at dawn of this day was not a dream. It had happened in reality.

Fairy Taya had indeed been here, and it was she who had left him an expensive birthday gift - six diamond stones, in memory of the first perfect number "six".

It was she who had given him his first math lesson, opening the door for him to a new world - the World of Numbers. He saw its presence in this room. There's a chandelier hanging on the ceiling. It's one. Under the chandelier stands a table. It's one. And around the table are chairs, exactly four of them. He remembered the numbers on himself and wiggled the fingers of his right hand. There are exactly five of them. And on the left hand, there are just as many - exactly five. Together, on both hands, there are ten fingers. The number "ten" is a very important number - the basis of counting. As fairy Taya taught, with its help, you can determine large numbers without knowing their names.

All these memories took Mosie several minutes, during which mom managed to change him into festive clothes: festive shorts and a T-shirt with short sleeves falling over them, with an inscription on the chest in Hebrew, which mom, dad, and Alice could read fluently, but Mosie didn't even know the alphabet. He was to study this in kindergarten, where he was going in the coming days, in accordance with the law on mandatory attendance of state kindergartens by children who have turned three years old. If Mosi could already read Hebrew, he would have read on his T-shirt an inscription which translates to Russian as: "Good boy of Waters of Wonder".

After dressing Mosie, mom took his hand and then led him through the room door downstairs. They began to descend, and Mosie unexpectedly began to count the steps: "One, two, three..."

There turned out to be exactly ten steps, and Mosie counted to the very last one. After all, fairy Taya had taught him to count to ten, and he hadn't forgotten.

When they came down to the first floor of the cottage, mom led Mosi to the door of the largest room in their house, where they celebrated holidays as a family, held Saturday meals, and received guests. She gently pushed the door, and as soon as it opened, a familiar melody with birthday greetings in English was heard from somewhere above:

Happy Birthday To You,
Happy Birthday To Mosie
Happy Birthday To You...

When they entered the room, Mosie saw that the large table, usually standing in its center, had been moved to the window and was already festively set, and in the center of the room in place of the table was a chair, around which stood dad, Alice, and Augusta with Thomas on her back. Dad and Alice are clapping their hands to the beat of the music, Augusta is wagging her tail and Thomas is echoing her. Mom joined dad and Alice, seating Mosie on the chair. Everyone began to circle around the chair while the festive melody continued to play.

When the sound ended, Augusta, dropping Thomas to the floor, jumped up and licked Mosie's face, which became the final chord of Mosie's birthday greetings from all his family members.

Now it was time for gifts. They were all already displayed on a low pedestal - a platform standing by the wall opposite the one where the table stood. Mosie was led to the platform, and he began to examine the gifts and get acquainted with them.

Dad gave him a talking constructor set that could give him tasks in dad's voice, guide and evaluate their completion, help correct mistakes. When Mosie, on dad's advice, pressed one of the buttons on the right panel of the constructor, it said in dad's voice: "Let's try!" and laughed with dad's laughter.

Mom gave Mosie a real car, only a small red one with an open top. It had a seat for the driver and a steering wheel for control, but at the same time, it had a built-in safety system that prevented the car, under any control, from hitting obstacles and falling into holes. Mosie decided that he would try out the car today.

The gift from Alice was as if it had been ordered by fairy Taya. It was a seven-digit abacus. The units of each digit were in the form of beads of one color, strung on one rod, the colors of the seven digits corresponded to the seven colors of the rainbow. The beads representing units on the lower rod were red. There were exactly ten of them. The beads representing tens, on the second rod, were all orange. There were also exactly ten of them. The beads on the third rod were yellow, on the fourth rod green, on the fifth - light blue, on the sixth - blue, on the seventh - purple.

On each of the rods there were exactly ten beads, with the help of this abacus one could represent, in the decimal system, all possible numbers, up to millions, and perform four arithmetic operations with them.

The abacus could be positioned vertically, so that the rod with red beads was at the bottom, or horizontally, so that this rod was the rightmost.

Mosie had yet to learn all of this.

Additionally, we note although this abacus looked as an ordinary one and similar to already existing abacuses, it had a special feature indicating that it was custom-made. A musical device was inserted into the frame to which the rods were attached. When moving each bead, this device produced a sound corresponding to one of the notes of the octave. In turn, each note corresponded to one of the colors of the rainbow.

The red color corresponded to the note "do", orange - "re", yellow - "mi", green - "fa", light blue - "sol", blue - "la", purple - "si". Thus, the abacus embodied all the colors of the rainbow and all the notes of the musical octave. A professional designer had worked on it by personal order.

When numbers were reproduced on this abacus and operations were performed on them, a musical melody was simultaneously created and played. Note that if desired or necessary, the musical device could be turned off, and then the abacus became an ordinary seven-digit abacus, only not monochrome, but multicolored.

On a separate stand above all the gifts towered a large shell, a gift from Augusta and Thomas. Now it wasn't glowing because the room was bright. Mosie remembered from mom's story that in the dark, rays of light emanate from the shell. He thought that they would remind him of the magic ray that came from under fairy Taya's nail. He immediately decided that he would keep the shell on the table in his room, putting it above the six diamond stones gifted to him by fairy Taya. Of all the wonderful gifts from his family, Mosie considered fairy Taya's gift the best.

When he sees the rays coming from the shell, he will remember fairy Taya and the new huge world she opened for him, surrounding him and everything in the Universe - the World of Numbers, world which is vivid, active and speaking to those who

understand its language. Mosie has yet to study this world, but he is ready for it, he desires it, and desire is the key to success. Happy birthday, Mosie!

Bon voyage, go studying the inexhaustible mysteries of the Universe, from our galaxy to Ephraxia.

Afterword: A Message to Our Readers

Our fairy tale, written for you, our dear little readers, has come to an end. Thank you for being with us. Our special thanks to the two children who inspired us to write it – a girl named Alice and a boy named Alexander.